Closing Wed: HW_4A, 4B, 4C (6.4, 6.5)

6.4 Work (continued)

Entry Task:

A cable with density 4 lbs/ft is being used to lift a 50 pound weight from the ground to the top of a 25 foot building. Find the total work done.

Step 1: Draw a picture.

Step 2: Break up the problem:

(a) Work to lift the 50 lbs weight?

(b) Work to lift the cable?

Step 3: Add these together.

Example:

You are pumping water out of an aquarium. The aquarium is a rectangular box with a base of 2 ft by 3 ft and height of 10 ft.

The density of water is 62.5 lbs/ft^3.

If the tank starts full, how much work is done in pumping all the water to the top and out over the side?

Quick Summary:

$$\text{Work} = \lim_{n\to\infty} \sum_{i=1}^{n} (FORCE)(DIST)$$

$$= \int_a^b (FORCE)(DIST)$$

Problem type 1: (Leaky bucket/spring)

Leaking at constant rate → f(x) = mx+b

Spring (Hooke's Law) → f(x) = kx

Force given → f(x) = force

FORCE = $f(x_i)$, DISTANCE = Δx

WORK = $\int_a^b f(x)dx$

Problem type 2: (Chain/pumping)

For all these,

FORCE = weight of a horizontal slice

DIST = distance moved by that slice

For chain:

k = density = force per distance

FORCE = weight of slice = $k\Delta x$

DIST = distance moved by slice

(typically *x* if you label like me)

WORK = $\int_0^b x\, kdx$

For pumping:

k = density = weight per volume

FORCE = k volume = k(hor. slice area)Δy

DIST = distance moved by slice

(typically *a-y* if you label like me)

WORK = $\int_0^b (a-y)k(slice\ area)dy$

Some unit facts:

g = grams, in = inches,

yd = yards, mi = miles

1000 g = 1 kg

100 cm = 1 meter

12 inches = 1 foot

3 feet = 1 yard

5280 ft = 1 mile

Density of water

= 1000 kg/m^3 = 9800 N/m^3

= 62.5 lbs/ft^3

	Metric	**Standard**
Mass	kg	
Accel.	9.8 m/s^2	32 ft/s^2
Force	Newtons N = kg·m/s^2	pounds = lbs
Dist.	m = meters	ft = feet
Work	Joules J = N·m	foot-pounds ft·lbs

Review: Particular scenarios

Type 1 Problems:

1. HW 4A/1, 2, 8, 9 and HW 4B/1
   Given force, just need to integrate!

$$\text{Work} = \int_a^b f(x)\,dx$$

2. HW 4A/3, 4 (Springs)
   (i) Covert all to meters
   (ii) Label natural length, *L*, and note that *L* corresponds to x = 0.

Force = f(x) = kx

$$\text{Work} = \int_a^b kx\,dx$$

Step 1: Find k
Step 2: Answer question.

Type 2 Problems:

FORCE = weight of a horizontal slice,
DISTANCE = distance to top

3. HW 4A/5 and HW 4B/2 (Chain)
   (i) k = density of chain = weight/dist
   (ii) FORCE at a subdivision = kΔx
   (iii) Label DIST to top.

$$\text{Work} = \int_a^b Dist \cdot\ kdx$$

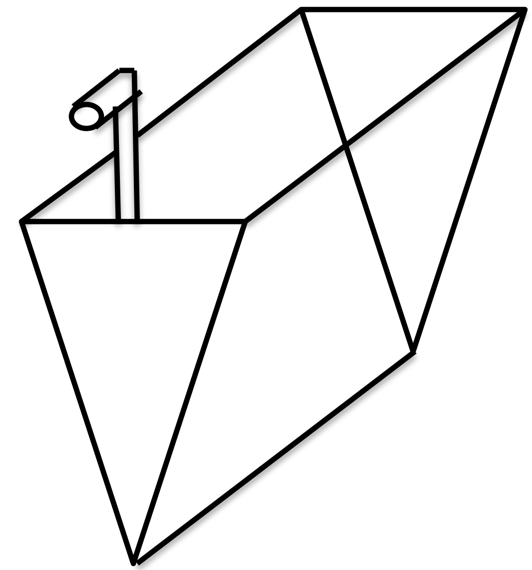
4. HW 4A/6,7 and HW 4B/3 (Pumping)

Water density = 9800 N/m^3 = 62.5 lbs/ft^3

(i) Label (put in xy-plane)
(ii) Draw a horizontal slice and find a formula for its area.
(iii) FORCE = (Density)(Area)Δy
(iv) DIST = distance to top

$$\text{Work} = \int_a^b (\text{Dist})(\text{Density})(\text{Area})\ \text{dy}$$

Example:
Consider the tank show at right.
The height is 3 meters, the width at the top is 2 meters and the length is 6 meters. Also we are pumping the water up to 4 meters above the ground (1 meter above the top edge).
If it starts full, how much work is done to pump it all out?



6.5 Average Value

The average value of the *n* numbers:

$$y_1, y_2, y_3, \ldots, y_n$$

is given by

$$\frac{y_1+y_2+y_3+\cdots+y_n}{n} = y_1\frac{1}{n} + \cdots + y_n\frac{1}{n}.$$

Goal: We want the average value of **all** the y-values of some function y = f(x) over an interval x = a to x = b.

Derivation:

1. Break into *n* equal subdivisions

   $\Delta x = \frac{b-a}{n}$, which means $\frac{\Delta x}{b-a} = \frac{1}{n}$

2. Compute *y*-value at each tick mark

   $y_1 = f(x_1), y_2 = f(x_2), \ldots, y_n = f(x_n)$

3. Ave $\approx f(x_1)\frac{\Delta x}{b-a} + \cdots + f(x_n)\frac{\Delta x}{b-a}$

   $$\text{Average} \approx \frac{1}{b-a}\sum_{i=1}^{n} f(x_i)\Delta x$$

4. Thus, we can define

   $$\text{Average} = \frac{1}{b-a}\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

5. Which means the exact average y-value of y = f(x) over x = a to x = b is

   $$f_{ave} = \frac{1}{b-a}\int_a^b f(x)dx$$